EXERCISES [MAI 5.1]

THE CONCEPT OF THE LIMIT

SOLUTIONS

Compiled by: Christos Nikolaidis

A. Paper 1 questions (SHORT)

1. (a)

For positive values of x near 0 (i.e. 0^+)	
x	$f(x)$
0.1	1.0517
0.01	1.0050
0.001	1.0005

For negative values of x near 0^-(i.e. 0^-)	
x	$f(x)$
– 0.1	0.9516
– 0.01	0.9950
– 0.001	0.9995

(b) $\lim_{x\to 0}\frac{e^x-1}{x}=1$

2. (a) $f(1)=0.2838$

(b) because the denominator is 0.

(c)

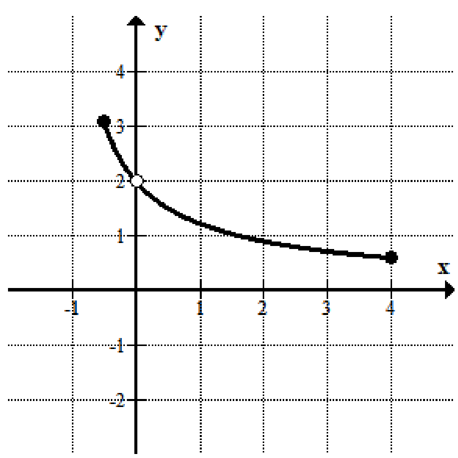
For values of x near 2^+:	
x	$f(x)$
2.1	0.5171
2.01	0.5017
2.001	0.5002

For values of x near 2^-	
x	$f(x)$
1.9	0.4837
1.99	0.4983
1.999	0.4998

(d) $\lim_{x\to 2}\frac{e^{x-2}-x+1}{(x-2)^2}=0.5$

3. (a) (i) 3.09 (ii) 1.23 (ii) 0.598

(b)



(c) $\lim_{Q\to 0}P=2$.

4. (a)

x	$f(x)$
100	2.87619
1 000	2.98706
1 000 000	2.99999

(b) $\lim_{x\to\infty}\frac{3x+2}{x+5}=3$

(c)

x	$f(x)$
- 100	3.13684
- 1 000	3.01307
- 1 000 000	3.00001

(d) $\lim_{x\to-\infty}\frac{3x+2}{x+5}=3$

(e) $\lim_{x\to\pm\infty}\frac{3x+2}{2x+5}=1.5$

5. (a) (i) $\lim_{x\to 3}\frac{x+3}{x-2}=6$ (ii) $\lim_{x\to+\infty}\frac{x+3}{x-2}=1$ (iii) $\lim_{x\to-\infty}\frac{x+3}{x-2}=1$

(b) (i) $\lim_{x\to 2^+}\frac{x+3}{x-2}=+\infty$ (ii) $\lim_{x\to 2^-}\frac{x+3}{x-2}=-\infty$

(iii) $\lim_{x\to 2^+}\frac{x-3}{x-2}=-\infty$ (iv) $\lim_{x\to 2^-}\frac{x-3}{x-2}=+\infty$

6. (a) (i) $\lim_{h\to 0}\frac{(2+h)^2-2^2}{h}=4$, $m_2=4$

(ii) $\lim_{h\to 0}\frac{(3+h)^2-3^2}{h}=6$, $m_3=6$

(b) (i) $m_5=10$, (ii) $m_{-5}=-10$

(c) $m_a=2a$

7. (a) (i) $\lim_{h\to 0}\frac{\ln(2+h)-\ln 2}{h}=0.5$, $m_2=0.5=\frac{1}{2}$

(ii) $\lim_{h\to 0}\frac{\ln(5+h)-\ln 5}{h}=0.2$, $m_3=0.2=\frac{1}{5}$

(b) $m_5=0.1=\frac{1}{10}$

(c) $m_a=\frac{1}{a}$

8. (a) $e^2 \cong 7.3891$

(b)

n	$\left(1+\frac{2}{n}\right)^n$
1	3.0000
10	6.1917
100	7.2446
1 000	7.3743
1 000 000	7.3890

(c) $\lim_{n\to+\infty}\left(1+\frac{2}{n}\right)^n = e^2$

(d) $\lim_{n\to+\infty}\left(1+\frac{3}{n}\right)^n = e^3$

B. Paper 2 questions (LONG)

9. (a)

Domain of f : $x \in [-1, 8[$	Range of f : $y \in [1, 4]$

(b)

$f(-1) = 2$	$f(1) = 4$	$f(4) = 2$	$f(6) = 2$	$f(8)$ not defined

(c)

$\lim_{x\to 1^-} f(x) = 2$	$\lim_{x\to 1^+} f(x) = 2$	$\lim_{x\to 1} f(x) = 2$
$\lim_{x\to 4^-} f(x) = 2$	$\lim_{x\to 4^+} f(x) = 2$	$\lim_{x\to 4} f(x) = 2$
$\lim_{x\to 6^-} f(x) = 4$	$\lim_{x\to 6^+} f(x) = 2$	$\lim_{x\to 6} f(x)$ does not exist

10. In fact, it is the same function as in exercise 9.

(a) and (b) as in exercise 9

(c)

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-}(x^2+1) = 2$
$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} 2 = 2$
$\lim_{x\to 1} f(x) = 2$
$\lim_{x\to 4^-} f(x) = \lim_{x\to 4^-} 2 = 2$
$\lim_{x\to 4^+} f(x) = \lim_{x\to 4^+}(x-2) = 2$
$\lim_{x\to 4} f(x) = 2$
$\lim_{x\to 6^-} f(x) = \lim_{x\to 6^-}(x-2) = 4$
$\lim_{x\to 6^+} f(x) = \lim_{x\to 6^+} 2 = 2$
$\lim_{x\to 6} f(x)$ does not exist